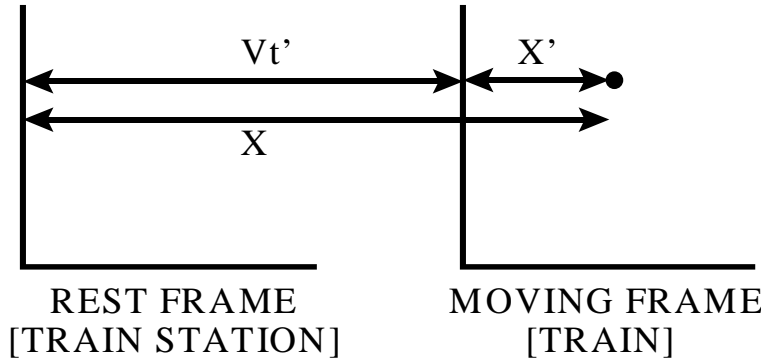


SPECIAL THEORY OF RELATIVITY

EINSTEIN'S POSTULATES

1. The speed of light is the same for all observers in all inertial frames of reference.
2. The laws of physics as we know them are the same for all inertial frames of reference.



$$x = K(x' + V \cdot t') \quad \text{and} \quad x' = K \cdot (x - V \cdot t)$$

$$x' = K \cdot [K \cdot (x' + V \cdot t') - V \cdot t] = K^2 \cdot x' + K^2 \cdot V \cdot t' - K \cdot V \cdot t$$

Solve for $K \cdot V \cdot t$

$$K \cdot V \cdot t = (K^2 \cdot V \cdot t' + K^2 \cdot x') - x' = x' \cdot (K^2 - 1) + K^2 \cdot V \cdot t'$$

$$K \cdot V \cdot t = x' \cdot (K^2 - 1) - K^2 \cdot V \cdot t'$$

$$t = \frac{x' \cdot (K^2 - 1) - K^2 \cdot V \cdot t'}{(K \cdot V)} = \frac{x' \cdot (K^2 - 1)}{(K \cdot V)} + K \cdot t'$$

By definition the velocity of a particle is given by

$$v = \frac{\Delta x}{\Delta t} = \frac{(x_2 - x_1)}{(t_2 - t_1)} = \frac{K \cdot (x'_2 + V \cdot t'_2) - K \cdot (x'_1 + V \cdot t'_1)}{\left[\frac{x'_2 \cdot (K^2 - 1)}{(K \cdot V)} + K \cdot t'_2 \right] - \left[\frac{x'_1 \cdot (K^2 - 1)}{(K \cdot V)} + K \cdot t'_1 \right]}$$

Divide both the numerator and denominator by K:

$$v = \frac{(x'_2 + V \cdot t'_2) - (x'_1 + V \cdot t'_1)}{\left[\frac{x'_2 \cdot (K^2 - 1)}{(K^2 \cdot V)} + t'_2 \right] - \left[\frac{x'_1 \cdot (K^2 - 1)}{(K^2 \cdot V)} + t'_1 \right]}$$

Re-group the numerator and denominator

$$v = \frac{(x'_2 - x'_1) + V \cdot (t'_2 - t'_1)}{\left(\frac{K^2 - 1}{K^2 \cdot V} \right) \cdot (x'_2 - x'_1) - (t'_2 - t'_1)}$$

Divide both the numerator and the denominator by Δt

$$v = \frac{\frac{(x'_2 - x'_1)}{(t'_2 - t'_1)} + V \cdot \frac{(t'_2 - t'_1)}{(t'_2 - t'_1)}}{\left(\frac{K^2 - 1}{K^2 \cdot V} \right) \cdot \frac{(x'_2 - x'_1)}{(t'_2 - t'_1)} - \frac{(t'_2 - t'_1)}{(t'_2 - t'_1)}}} = \frac{\frac{(x'_2 - x'_1)}{(t'_2 - t'_1)} + V}{\left(\frac{K^2 - 1}{K^2 \cdot V} \right) \cdot \frac{(x'_2 - x'_1)}{(t'_2 - t'_1)} - 1}$$

but remember that $\frac{(x'_2 - x'_1)}{(t'_2 - t'_1)}$ is equal to v' and therefore

$$v = \frac{v' + V}{\left(\frac{K^2 - 1}{K^2 \cdot V} \right) \cdot c - 1}$$

But one of Einstein's postulates was that the speed of light is the same whether the observer is in the rest frame or in the moving frame so for light $v = v' = c$!

$$c = \frac{c + V}{\left(\frac{K^2 - 1}{K^2 \cdot V} \right) \cdot c - 1} \quad \text{becomes} \quad \left[\frac{(K^2 - 1)}{K^2 V} \cdot c - 1 \right] \cdot c = c + V$$

then $\frac{(K^2 - 1)}{K^2 V} \cdot c^2 - c = V$ and $(K^2 - 1) \cdot c^2 = K^2 \cdot V^2$

and then $K^2 \cdot c^2 - c^2 = K^2 \cdot V^2$ $K^2(c^2 - V^2) = c^2$

solving for K $K = \sqrt{\frac{c^2}{(c^2 - V^2)}} = \sqrt{\frac{1}{\frac{(c^2 - V^2)}{c^2}}} = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$ $K = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$

RELATIVISTIC LENGTH CONTRACTION

Remember from the beginning that

$$x = K(x' + V \cdot t')$$

The length of an object is determined by taking the difference between the two x coordinates:

$$L_o = x_2 - x_1 = (K(x'_2 + V \cdot t'_2) - K(x'_1 + V \cdot t'_1)) = \frac{[(x'_2 + V \cdot t'_2) - (x'_1 + V \cdot t'_1)]}{\sqrt{1 - \frac{V^2}{c^2}}}$$

Reorganize the terms so that the x's are together and the t's are together:

$$L_o = \frac{(x'_2 - x'_1)}{\sqrt{1 - \frac{V^2}{c^2}}} + \frac{V \cdot (t'_2 - t'_1)}{\sqrt{1 - \frac{V^2}{c^2}}}$$

But to measure the length of an object you must measure both x's at the same time therefore the t's must be the same!

Therefore

$$L_o = \frac{(x'_2 - x'_1)}{\sqrt{1 - \frac{V^2}{c^2}}}$$

The difference $x'_2 - x'_1$ is equal to the length L of the object in the moving frame of reference and so therefore:

$$L_o = \frac{L}{\sqrt{1 - \frac{V^2}{c^2}}}$$

and finally cross multiplying

$$L = L_o \cdot \sqrt{1 - \frac{V^2}{c^2}}$$

where L_o is the length of the object as measured when the observer is at rest relative to the object and where L is the length measured when the observer is moving with a velocity V relative to the object.

RELATIVISTIC TIME DILATION

Likewise, the time transformation between the rest frame and the moving frame of reference is given by:

$$t = \frac{x' \cdot (K^2 - 1)}{(K \cdot V)} + K \cdot t'$$

As before the length of a time interval is equal to the difference in two different times:

$$T_0 = t_2 - t_1 = \left[\frac{x'_2 \cdot (K^2 - 1)}{(K \cdot V)} + K \cdot t'_2 \right] - \left[\frac{x'_1 \cdot (K^2 - 1)}{(K \cdot V)} + K \cdot t'_1 \right]$$

Again reorganizing terms so as to group the x's and the t's separately:

$$T_0 = K \cdot (t'_2 - t'_1) + (x'_2 - x'_1) \cdot \frac{(K^2 - 1)}{K \cdot V}$$

But just as before, it is unnecessary to change your position while measuring the time interval

$$x'_2 - x'_1 = 0 \quad \text{and so the equation becomes} \quad T_0 = K \cdot (t'_2 - t'_1)$$

Again replacing K and recognizing that $t'_2 - t'_1$ is the length of the time interval T that passes in the moving frame of reference. Finally giving:

$$T_0 = \frac{T}{\sqrt{1 - \frac{V^2}{c^2}}} \quad \text{or} \quad T = T_0 \cdot \sqrt{1 - \frac{V^2}{c^2}}$$

where: T_0 is the time that passes in the rest frame, T is the time that passes in the moving frame, and where V is the velocity if the observer relative to the system being observed.

RELATIVISTIC VELOCITY TRANSFORMATIONS

From before it was shown that the relationship between the velocity measure in the rest frame vs the velocity measure from the moving frame is given by:

$$v = \frac{v' + V}{1 + \frac{(K^2 - 1)}{K^2 \cdot V} \cdot v'}$$

If we isolate the part of the denominator that includes K and simplify:

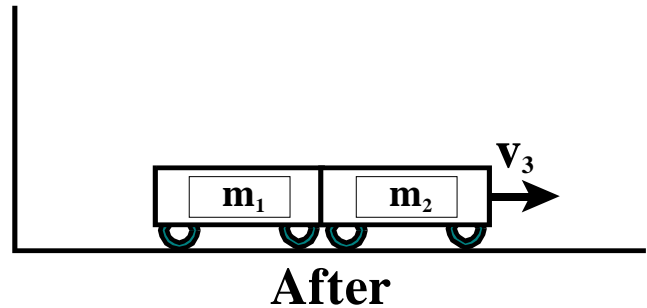
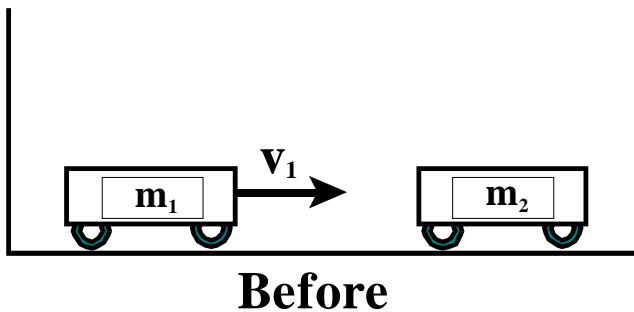
$$\frac{K^2 - 1}{K^2 \cdot V} = \frac{\left(\frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}\right)^2 - 1}{\left(\frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}\right)^2 \cdot V} = \frac{\frac{c^2}{(c^2 - V^2)} - 1}{\frac{c^2}{(c^2 - V^2)} \cdot V} = \frac{\frac{c^2 - (c^2 - V^2)}{c^2 - V^2}}{\left(\frac{c^2 \cdot V}{c^2 - V^2}\right)} = \frac{V^2}{c^2 \cdot V} = \frac{V}{c^2}$$

Replacing $\frac{K^2 - 1}{K^2 \cdot V}$ with $\frac{V}{c^2}$ you get: $v = \frac{v' + V}{1 + \frac{V}{c^2} \cdot v'}$ the resulting velocity transformation!

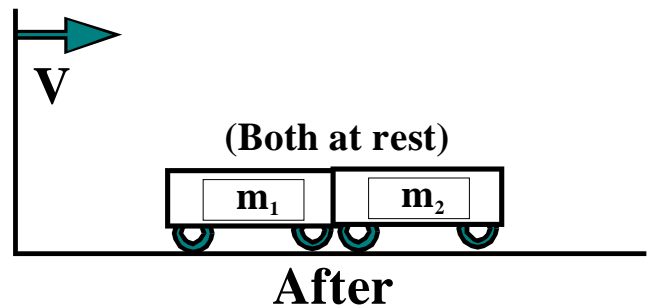
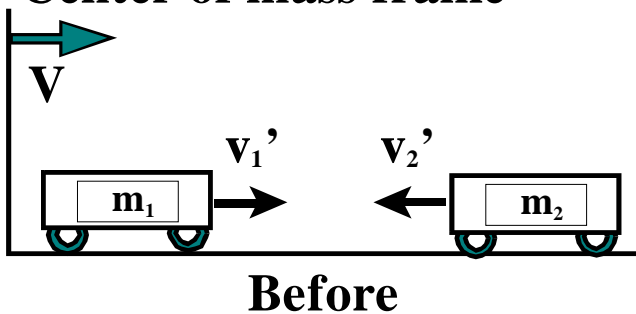
RELATIVISTIC MASS EXPANSION

One of the so called relativistic invariants is momentum conservation. What this means is that if momentum is conserved in one inertial frame of reference, it must also be conserved when observed from any other inertial frame of reference. To do this consider a collision where two objects, whose masses are identical when measured at rest in the same frame of reference, are observed to collide from two different frames: the rest frame and the center of mass frame of reference.

Rest frame



Center of mass frame



In the rest frame cart m_1 is initially moving toward the right with a velocity v_1 when it collides with cart m_2 which is initially at rest. After the collision the two cars stick together and move off to the right with a reduced velocity v_3 . When this same collision is observed from the center of mass frame of reference cart m_1 is initially moving to the right with a velocity v_1' while cart m_2 is moving to the left with a velocity of v_2' . After the collision both carts are at rest in the center of mass frame of reference.

Since the carts after are moving to the right with a velocity v_3 in the rest frame while they are at rest after in the center of mass frame, the velocity of the center of mass frame V and the final velocity in the rest frame v_3 must be the same.

Likewise, since the velocity of m_2 was initially zero in the rest frame while it is v_2' in the center of mass frame, the velocity in the center of mass frame must be due exclusively to the motion of the center of mass frame. Therefore, v_2' must also be equal to the velocity of the center of mass frame V .

Finally, if the total momentum of the system in the center of mass frame is zero after the collision it must also be zero before the collision. This can only be true if the momentum's of m_1 and m_2 are equal but opposite before and after. Assuming that both masses, m_1 and m_2 , have the same mass then v_1' must equal v_2' . But since we have already shown that v_2' is equal to V we come to the conclusion that $v_1'=v_2'=v_3=V$. All v 's in the problem, except for v_1 , are equal to V , the velocity of the center of mass frame of reference.

RELATIVISTIC MASS EXPANSION - CONTINUED

Now previously we have shown that the velocity between two different inertial systems of reference can be transformed through:

$$v = \frac{v' + V}{1 + \frac{v' \cdot V}{c^2}}$$

But because of the arguments above we have shown that all of the velocities measured from the center of mass frame of reference are equal to the velocity of the center of mass frame of reference V . Because of this the velocity transformation equation becomes:

$$v = \frac{2V}{1 + \frac{V^2}{c^2}} \quad (1)$$

Because of momentum conservation the collision in the rest frame of reference can be described by:

$$m_1 \cdot v_1 = (m_1 + m_2) \cdot v_3 = (m_1 + m_2) \cdot V \quad (2)$$

where v_3 is equal to V as shown previously. Since there is only one velocity remaining in the rest frame v_1 , we will drop the subscript and use v for v_1 .

The next step will be to substitute equation (1) into equation (2).

$$m_1 \cdot \left(\frac{2V}{1 + \frac{V^2}{c^2}} \right) = (m_1 + m_2) \cdot V$$

Multiply both the numerator and denominator of (1) by c^2 .

$$m_1 \cdot \left(\frac{2V \cdot c^2}{c^2 + V^2} \right) = (m_1 + m_2) \cdot V$$

Move both m_1 's to the same side of the equation.

$$m_1 \cdot \left(\frac{2V \cdot c^2}{c^2 + V^2} \right) - m_1 \cdot V = m_2 \cdot V$$

Find a common denominator.

$$m_1 \cdot \left(\frac{2V \cdot c^2}{c^2 + V^2} \right) - m_1 \cdot V \cdot \left(\frac{c^2 + V^2}{c^2 + V^2} \right) = m_2 \cdot V$$

Divide both sides by V .

$$m_1 \cdot \left(\frac{2 \cdot c^2}{c^2 + V^2} \right) - m_1 \cdot \left(\frac{c^2 + V^2}{c^2 + V^2} \right) = m_2$$

RELATIVISTIC MASS EXPANSION - CONTINUED

Combine terms on the left side.

$$m_1 \cdot \left(\frac{2 \cdot c^2 - c^2 - v^2}{c^2 + v^2} \right) = m_1 \cdot \left(\frac{c^2 - v^2}{c^2 + v^2} \right) = m_2$$

Put both m's on the left side of the equation and take the square root of the square on the right side.

$$\frac{m_2}{m_1} = \sqrt{\frac{(c^2 - v^2)^2}{(c^2 + v^2)^2}}$$

Expand the square of the numerator and complete the square.

$$\frac{m_2}{m_1} = \sqrt{\frac{c^4 - 2 \cdot c^2 \cdot v^2 + v^4}{(c^2 + v^2)^2}} = \sqrt{\frac{c^4 + 2 \cdot c^2 \cdot v^2 + v^4 - 4 \cdot c^2 \cdot v^2}{(c^2 + v^2)^2}}$$

Re-group the numerator and square.

$$\frac{m_2}{m_1} = \sqrt{\frac{(c^4 + 2 \cdot c^2 \cdot v^2 + v^4) - 4 \cdot c^2 \cdot v^2}{(c^2 + v^2)^2}} = \sqrt{\frac{(c^2 + v^2)^2 - 4 \cdot c^2 \cdot v^2}{(c^2 + v^2)^2}}$$

Which then becomes.

$$\frac{m_2}{m_1} = \sqrt{\frac{(c^2 + v^2)^2 - 4 \cdot c^2 \cdot v^2}{(c^2 + v^2)^2}} = \sqrt{1 - \frac{4 \cdot c^2 \cdot v^2}{(c^2 + v^2)^2}}$$

Divide both the numerator and the denominator by c^4 , simplify and then divide both the numerator and denominator by $(c^2 + v^2)$.

$$\frac{m_2}{m_1} = \sqrt{1 - \frac{\frac{4 \cdot c^2 \cdot v^2}{c^4}}{\frac{(c^2 + v^2)^2}{c^4}}} = \sqrt{1 - \frac{\frac{4 \cdot v^2}{c^2}}{\frac{(c^2 + v^2)}{c^4}}} = \sqrt{1 - \frac{4 \cdot v^2 \cdot c^2}{(c^2 + v^2)^2}}$$

Writing this term as a square.

$$\frac{m_2}{m_1} = \sqrt{1 - \left(\frac{2 \cdot v}{c^2 + v^2} \right)^2 \cdot c^2}$$

RELATIVISTIC MASS EXPANSION - CONTINUED

And then dividing both the numerator and the denominator of the fraction by c^2 .

$$\frac{m_2}{m_1} = \frac{\sqrt{1 - \left[\frac{\frac{2 \cdot v}{c^2}}{\left(\frac{c^2 + v^2}{c^2} \right)} \right]^2} \cdot c^2}{\sqrt{1 - \frac{\left(\frac{2 \cdot v}{c^2} \right)^2 \cdot c^2}{\left(1 + \frac{v^2}{c^2} \right)^2}}$$

If you pull out the c^2 from inside the brackets you get.

$$\frac{m_2}{m_1} = \frac{\sqrt{1 - \left(\frac{2 \cdot v}{1 + \frac{v^2}{c^2}} \right)^2} \cdot \frac{1}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

But what is contained within the parenthesis is the same as equation (1) = v !

Therefore $\frac{m_2}{m_1} = \sqrt{1 - \frac{v^2}{c^2}}$ solving for m_1 $m_1 = \left(\frac{m_2}{\sqrt{1 - \frac{v^2}{c^2}}} = K \cdot m_2 \right)$

where m_2 is the mass of the cart measured when it is at rest in the rest frame [m_0] and where m_1 is the mass of the cart when it is moving in the rest frame [m] and therefore:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

MASS - ENERGY EQUIVALENCE

The equation for mass expansion can be written:

$$m = m_0 \cdot \left(1 - \frac{v^2}{c^2} \right)^{\frac{-1}{2}} \quad (3)$$

According to the binomial expansion:

$$(A + B)^n = A^n + n \cdot A^{(n-1)} \cdot B + n \cdot (n-1) \cdot A^{(n-2)} \cdot \frac{B^2}{2} + \text{other}$$

Applying the binomial expansion to equation (3) above: $A = 1$ $B = \left(\frac{-v^2}{c^2} \right)$ $n := \frac{-1}{2}$

MASS - ENERGY EQUIVALENCE

Using the binomial expansion to expand the mass equation you get:

$$m = m_0 \cdot \left(1 - \frac{v^2}{c^2}\right)^{\frac{-1}{2}} = m_0 \cdot \left[1^{\frac{-1}{2}} + \frac{-1}{2} \cdot 1^{\frac{-3}{2}} \cdot \left(\frac{-v^2}{c^2}\right) + \frac{-1}{2} \cdot \frac{-3}{2} \cdot 1^{\frac{-5}{2}} \cdot \frac{\left(\frac{-v^2}{c^2}\right)^2}{2} + \text{other} \right]$$

Which becomes:

$$m = m_0 \cdot \left(1 + \frac{v^2}{2 \cdot c^2} + \frac{3}{8} \cdot \frac{v^4}{c^4} + \text{other}\right)$$

Arguing that all terms after the first two are diminishingly small this becomes:

$$m = m_0 \cdot \left(1 + \frac{v^2}{2 \cdot c^2}\right)$$

If this is expanded and the simplified:

$$m = m_0 + m_0 \cdot \frac{v^2}{2 \cdot c^2}$$

Multiply each term by c^2 :

$$m \cdot c^2 = m_0 \cdot c^2 + \frac{1}{2} \cdot m_0 \cdot v^2$$

Rearranging:

$$\frac{1}{2} m_0 \cdot v^2 = m \cdot c^2 - m_0 \cdot c^2$$

$$\text{KE} = \text{TE} - \text{RE}$$

Where $\frac{1}{2} m_0 v^2$ is the kinetic energy KE, mc^2 is the total energy TE, and $m_0 c^2$ is the rest energy RE

Finally:

$$\text{KE} = m \cdot c^2 - m_0 \cdot c^2 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot c^2 - m_0 \cdot c^2 = m_0 \cdot c^2 \cdot \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

Therefore: $\text{KE} = \text{RE} \cdot (\text{K} - 1)$

THIS MAY BE THE END!

