

What's a Fermi Question?

A "Fermi question" is a question in physics which seeks a fast, rough estimate of quantity which is either difficult or impossible to measure directly. Fermi Questions are named after Enrico Fermi, a Nobel Laureate in Physics, who was famous for doing order-of-magnitude calculations in his head.

Your students can use their 'powers' of estimation and using exponents to solve a problem that is difficult or impossible to solve exactly, by estimating to the nearest power of 10 ("order of magnitude").

No calculators or other devices, or reference materials allowed!

Why use Fermi Questions in your class?

- This is good for student math skills (exponents, rounding, mental math, unit analysis, assumptions).
- Fermi Questions are great for warm-ups/Do-Nows, competitions between students/teams, or substitute-day plans (do small group work, show justification for answer).
- It can be useful to discuss situations in which an exact answer is not needed, just an estimate, and that estimating is an important skill.
- Students also apply general knowledge – populations, distances, English-metric approximations, area/volume formulas, etc.

Example: How many times does the average person's heart beat in a lifetime? (No calculators!)

Estimating:

estimate average heart rate ≈ 60 beats/ min ~ 1 beat/sec

estimate average lifespan ≈ 80 years/lifetime

estimate # seconds in 1 year $\approx 3 \times 10^7$ s/yr here's how:

$$\begin{aligned} 365 \text{ days/y} \times 24 \text{ h/d} \times 3600 \text{ s/h} &\approx \frac{400 \text{ days} \times 20 \text{ h} \times 4000 \text{ s/h}}{4 \times 10^2 \times 2 \times 10^1 \times 4 \times 10^3} = \\ &32 \times 10^6 = 3.2 \times 10^1 \times 10^6 \approx \underline{3 \times 10^7 \text{ s/y}} \end{aligned}$$

Number of heartbeats in a lifetime:

$$(80 \text{ years}) \times (3 \times 10^7 \text{ s/year}) \times (1 \text{ beat/s}) = 240 \times 10^7 =$$

$$2.4 \times 10^2 \times 10^7 = \mathbf{2.4 \times 10^9} \approx \mathbf{10^9 \text{ heartbeats!}}$$

Just report the power of 10, not the coefficient (2.4).

Using "exact" numbers and calculator:

Number of heartbeats in a lifetime (using more "exact" numbers)

$$(77 \text{ years}) \times (3.15 \times 10^7 \text{ s/year}) \times (1 \text{ beat/s}) = 2.43 \times 10^9 \approx \mathbf{10^9 \text{ heartbeats!}}$$

(same as estimated w/o calculator!)

Scoring in Fermi Question competitions can vary (Science Olympiad, Physics Olympics, etc).

In the NJ Physics Olympics, each question is given a score of 10 points, minus 1 point for every power of 10 the team is off from the accepted answer. No score less than zero is given. Highest score wins! (So, in the example above, a team answer of 10^{11} would earn a score of 8 points).

Where to find Fermi Questions: NJAAPT website has questions from previous Physics Olympics. Also, just search online for "Fermi Questions" and you find many available online!

Note: there can be negative exponent problems (10^{-6}), but save them for later!

Here's one to try now: How many revolutions will a 14-inch radius tire have to make during a crossing of the Continental US?

Answer:

How many revolutions will a 14-inch radius tire have to make during a crossing of the Continental US?

$C=2\pi r$ Continental US \approx 3000 miles 1 mile = 5280 ft 14 inches \approx 1 ft

Estimate:

$$C = 2\pi r \approx 2(3)(1) = \underline{\underline{6 \text{ ft}}}$$

$$\text{Distance: } 3000 \text{ miles} \approx \text{___ ft} \quad 3000 \text{ mi} \times (5000 \text{ ft/mi}) = (3 \times 10^3) \times (5 \times 10^3) = \underline{\underline{\approx 15 \times 10^6 \text{ ft}}} = \text{total distance}$$

$$\# \text{ rev} = 15 \times 10^6 \text{ ft} \times (1 \text{ rev}/6 \text{ ft}) \approx 2 \times 10^6 \text{ revolutions.} \quad \text{Answer} \approx \underline{\underline{10^6}} \text{ revolutions}$$

Use more "exact" numbers:

$C=2\pi r$ Continental US \approx 3000 miles 1 mile = 5280 ft 14 inches = 1.16 ft

$$C = 2\pi r = 2(3.14)(1.16) = \underline{\underline{7.28 \text{ ft}}}$$

$$\text{Distance: } 3000 \text{ miles} = \text{___ ft} \quad 3000 \text{ mi} \times (5280 \text{ ft/mi}) = \underline{\underline{1.58 \times 10^7 \text{ ft}}} = \text{total distance}$$

$$\# \text{ rev} = 1.58 \times 10^7 \text{ ft} \times (1 \text{ rev}/7.28 \text{ ft}) = 2.17 \times 10^6 \text{ revolutions.} \quad \text{Answer} = \underline{\underline{10^6}} \text{ revolutions} \\ \text{(same as estimated!)}$$

FERMI QUESTIONS - Practice

1. How many piano tuners are there in Chicago?
2. What is pi to the power of 9?
3. How many pounds of rice were consumed by people living in the U.S. last year?
4. How many board game dice does it take to equal the mass of the average human?
5. What number of tennis balls would you have to lay on the surface area of an Olympic-sized swimming pool in order to fully cover it?
6. How much trash (in pounds) does the average family produce in a year?
7. What volume of air (in Liters) do you breathe in one day?
8. How many hairs are on the average human head (that actually has hair!)
9. If we could brew coffee in swimming pools, how many pools would we need to satisfy the need in the U.S. on a given day?
10. How many pennies would it take to make a stack the height of the Empire State Building?

(Cover up answers below, if you photocopy this for students to try!)

1. 10^2
2. 10^4
3. 10^{10}
4. 10^4
5. 10^5
6. 10^3
7. 10^3
8. 10^5
9. 10^2
10. 10^5

Notes to Teacher:

- Do some examples with students in class before giving the worksheet, so they get the idea.
- You can make up some of your own, too. (example: “what is the national debt per person in the US?” “How many compact cars can be placed end-to-end around the circumference of the earth?”).
- When using the worksheet above, remember to cover the answers before photocopying!
- Please do NOT post on the Internet, out of consideration of worksheet security for other teachers.
- The worksheet does not have any negative exponent examples, but they are important to do, once students get the basics of tackling Fermi Questions.
- This is also a good activity in which to discuss assumptions that are made in the examples, and assumptions in science/life in general.
- Have fun!

Regards, NancyM.